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**A vendor managed inventory scheme for supply chain coordination under
multiple heterogeneous retailers**

by

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ABSTRACT

This paper addresses the problem of supply chain coordination in the context of Vendor Managed Inventory (VMI) involving one supplier selling to multiple downstream heterogeneous retailers. The VMI contract considers a penalty scheme wherein, the retailers impose a per unit penalty cost on the supplier for exceeding the shipment from a pre-determined upper limit of inventory. The proposed model comes up with an optimal replenishment scheme wherein the supplier increases the shipment batch size such that none of the retailers are worse off. We compare the proposed replenishment model with joint economic lot size (JELS) and other VMI models, reflecting the improvement over those models in terms of costs. We also show the equivalence of the proposed replenishment model with JELS model having unequal reorder intervals.

Keywords: Vendor managed inventory; Operation management; multi-item joint replenishment; Inventory Control

1. Introduction and Literature Review

Coordination in supply chain is a well-researched area. A supply chain is said to be coordinated when the set of actions of all the partners involved forms a Nash Equilibrium [1]. In a supply chain involving a supplier and a retailer, the retailer would always prefer to place an order according to his EOQ (economic order quantity), which is his optimal strategy. However, such a strategy is suboptimal for the supplier as well as for the entire supply chain. In this context, coordination schemes like Joint Economic Lot Size (JELS) models ([2] and [3]) have been developed for single supplier – single retailer scenario. It has been observed that JELS models minimize the total cost of the entire supply chain. However, in JELS, not all the entities of the supply chain benefit equally and some might be worse off, forcing them to opt out of such a scheme ([4] and [5]). Hence the need to develop some decentralized coordination mechanisms to resolve this conflict. In this regard, various coordination mechanisms have been developed in the past by different researchers. Vendor Managed Inventory (VMI) is one such mechanism, which has gained prominence in the recent years.

VMI differs from traditional inventory management system in the sense that the inventory replenishment decision at the retailers' premises is taken by the supplier instead of the retailers [6]. VMI attained prominence after the partnership between Wal-Mart and Procter & Gamble became successful in 1985 [7]. A common form of VMI contract considers a penalty scheme, where the supplier is charged a penalty for every extra unit replenished to the retailers that exceed a pre-defined limit ([5], [8], [9], [10], and [11]).

There have been several studies trying to address the issue of supply chain coordination through VMI. Studies like [5], [10], [12] and [13] have obtained centralized solution for the supply chain. Chakraborty et al. [11] extended the scope of coordination models under VMI to find Pareto Optimal solution for the Supply Chain coordination problem, where the retailer will not be made worse off from his initial optimum point. The penalty from the supplier to the retailer acts as a form of compensation or side-payment to incentivize the retailer to accept shipments in larger batch sizes.

While the JELS models have not been extended to handle multiple retailers, the VMI models in the literature have been developed for both single supplier-single retailer ([11], [12], and [14]) as well as single supplier –multiple retailers ([5], [15], and [16]). The latter type of models can be further divided into two categories: (a) Models with all retailers having equal reorder interval (ERI) [5], and (b) models with retailers having unequal reorder interval (URI) ([10], [17] and [18]). For the general case where the retailers are heterogeneous, the URI models have been shown to perform better than the ERI models.

In this paper, we develop and solve a single supplier-multiple retailers VMI model with URI, where the supplier decides on a replenishment policy to minimize his own cost, without making any of the retailers worse off. The proposed model is essentially an extension of the single supplier-single retailer VMI model developed by Chakraborty et al. [11].

We also develop and solve the JELS models for single supplier and multiple retailers with both ERI and URI. With the help of an example we show how the proposed model performs better than the VMI model with ERI [5], VMI model with URI [10], JELS model with ERI and JELS model with URI. We also show analytically the equivalence of the proposed model with that of

the JELS model with URI. Further, with the help of the sensitivity analysis we show the robustness of the proposed model as compared to the other models.

The paper is organized as follows: section 2 presents the mathematical model proposed in this paper, section 3 covers JELS models involving a single supplier supplying to multiple retailers, section 4 provides the equivalence of our proposed model with JELS model under URI, section 5 presents the numerical analysis along with sensitivity analysis, and lastly section 6 concludes the paper while highlighting the contributions made to the literature.

2. Mathematical Model

2.1. Problem Statement

We consider a setting wherein a supplier is supplying a product to a number of retailers. The retailers are different from one another in the sense that their respective demand, ordering costs and inventory carrying costs are different. The supplier has a VMI contract with each one of the retailers and wishes to come up with an optimal replenishment policy for them.

Prior to the VMI contract, it's optimal for every retailer to order according to its EOQ. A retailer agrees to enter in a VMI contract, only after the supplier proposes a replenishment scheme such that the retailer is not worse off as compared to his earlier cost. Hence, in our proposed model, we seek to provide a Pareto optimal solution where the supplier proposes a replenishment policy such that, it minimizes his own cost and at the same time ensures none of the retailers are worse off as compared to their earlier cost. This Pareto optimal solution is unlike the centralized solution approaches of Darwish and Odah [5], Yao et al. [12] & Dong and Xu [19].

Moreover, in this work the upper limit for every retailer is also found out endogenously i.e. the penalty cost given to every retailer gets decided within the model. In this paper we assume that supplier is capable of cross-docking and hence does not carry any inventory. Such an assumption is the part of existing literature ([3], [11], and [20]).

2.2. Notations

2.2.1. Data

N No. of retailers

D_i	Annual demand of the i^{th} retailer
D	Cumulative demand ($D = \sum_{i=1}^N D_i$)
C_{oi}	Ordering cost of the i^{th} retailer
C_{li}	Inventory holding cost of the i^{th} retailer
C_S	Setup cost of the supplier
Q_i	EOQ corresponding to the i^{th} retailer
x_i	Per unit penalty charged by the i^{th} retailer

2.2.2. Variables

τ	Base replenishment cycle
z_i	Upper limit of inventory for the i^{th} retailer
M_i	Integer corresponding to the replenishment cycle of i^{th} retailer
q_i	Replenishment quantity for the i^{th} retailer
P_i	Penalty paid to the i^{th} retailer
y_i	Binary variable denoting whether the replenished quantity has exceeded the upper limit (z_i) for retailer i or not.
S	Supplier's total cost
B_{1i}	i^{th} retailer's total cost when acting independently
B_{2i}	i^{th} retailer's total cost after the VMI implementation

2.3. Assumptions

- (i) Final demand is deterministic
- (ii) No backordering is allowed
- (iii) Supplier has an unlimited capacity

- (iv) Transportation costs have not been considered separately and will be included in the ordering costs
- (v) Supplier doesn't hold any inventory

2.4. Development of the Mathematical Model

Each of the ' N ' retailers when acting independently place orders according to their respective economic order quantities (EOQ) which is given by

$$Q_i = \sqrt{\frac{2D_i C_{oi}}{C_{li}}} \quad (1)$$

while their respective cost is given as: $B_{li} = \frac{D_i}{Q_i} C_{oi} + \frac{Q_i}{2} C_{li} = \sqrt{2D_i C_{oi} C_{li}}$ (2)

Under VMI, the supplier has to decide on the optimal replenishment quantities for the retailers. He decides on a base replenishment cycle denoted by ' τ ' and ships the retailers after every ' $M_i \cdot \tau$ ' time where M_i 's are the integers such that for at least one i , $M_i = 1$. Whenever the supplier replenishes anything that exceeds the upper limit z_i , he pays a penalty P_i to the retailer ' i '. The penalty scheme helps in enticing the retailers to carry higher inventory. Moreover, while replenishing, the supplier also keeps in mind that the penalty scheme has to be designed in such a way, that none of the retailers is made worse off. In the presence of VMI under penalty scheme, the respective costs are given as:

$$\text{Cost of the supplier } S = \frac{C_s}{\tau} + \sum_{i=1}^N P_i \quad (3)$$

$$\text{Cost of } i^{\text{th}} \text{ retailer } B_{2i} = \frac{D_i}{q_i} C_{oi} + \frac{q_i}{2} C_{li} - P_i \quad (4)$$

$$\text{Replenishment quantity for } i^{\text{th}} \text{ retailer, } q_i = D_i M_i \tau \quad (5)$$

Penalty cost P_i for retailer i can be defined as :-

$$P_i = p_i \cdot y_i \quad (6)$$

Where,

$$p_i = \frac{x_i (q_i - z_i)^2}{2q_i}$$

$$y_i = \begin{cases} 1 & \text{if } q_i > z_i \\ 0 & \text{otherwise} \end{cases}$$

The supplier's objective is to minimize his costs post VMI implementation so that none of the retailers are made worse off. Hence the supplier's optimization problem is given as:

$$\text{Min } \frac{C_s}{\tau} + \sum_{i=1}^N y_i p_i \quad (7)$$

S.T.

- i. $\frac{C_{oi}}{M_i \tau} + \frac{D_i M_i \tau C_{li}}{2} - y_i p_i \leq \frac{D_i}{Q_i} C_{oi} + \frac{Q_i}{2} C_{li} \quad \forall i$
- ii. $D_i M_i \tau \geq y_i z_i \quad \forall i$
- iii. $z_i \geq (1 - y_i) D_i M_i \tau \quad \forall i$
- iv. $M_i \geq 1 \quad \forall i$
- v. $y_i \in [0, 1] \quad \forall i$
- vi. $\tau > 0, z_i > 0 \quad \forall i$

Constraint (i) is the individual rationality constraint for each of the retailers indicating that a retailer will not participate in the collaborative agreement if it makes him worse off as compared to the initial case when he was ordering according to his own EOQ. Constraints (ii) and (iii) indicate that the penalty will be paid to the i^{th} retailer only if the supplier's shipment exceeds the upper limit z_i .

3. Other Multi-Retailer Replenishment Models

In this section, we compare our proposed model with some of the previously studied models on supply chain coordination involving single supplier multiple retailers, and show using numerical examples that the proposed model perform better in terms of cost savings.

3.1. JELS model with ERI

In JELS models, there is a central decision maker who aims for global optimal solution for the supply chain. Under JELS model with ERI, the supplier produces Q_s units in every cycle, and replenishes q_i units to retailer i in every such cycle i.e. $Q_s = \sum_i q_i$.

$$\text{Also, } \frac{Q_s}{D} = \frac{q_i}{D_i} \quad \forall i \quad [\text{Equal replenishment cycle}]$$

The total supply chain cost (to be minimized) can then be written as:-

$$TSC = \frac{D}{Q_s} C_s + \frac{D}{Q_s} \sum_i C_{oi} + \frac{1}{2} \sum_i q_i h_i = \frac{D}{Q_s} C_s + \frac{D}{Q_s} \sum_i C_{oi} + \frac{1}{2} \sum_i Q_s \frac{D_i}{D} h_i \quad \text{using } \frac{Q_s}{D} = \frac{q_i}{D_i}$$

Differentiating above with respect to Q_s gives the following optimal values:

$$Q_s^* = \sqrt{\frac{2D^2 \left(C_s + \sum_i C_{oi} \right)}{\sum_i D_i C_{1i}}} \quad \text{and } q_i = \frac{D_i}{\sum_i D_i} Q_s^*$$

Second order condition gives $\frac{2D}{Q_s^3} C_s + \frac{2D}{Q_s^3} \sum_i C_{oi} > 0$ for $Q_s > 0$ and hence Q_s^* gives the optimal solution.

3.2. JELS model with URI

Under such a setting the supplier doesn't restrict himself to a common replenishment cycle and selects replenishment period given by $M_i \tau$ where M_i are integers and τ is the base replenishment cycle. The objective function can be written as given below:

$$\text{Min}_{M_i, \tau} TSC = \frac{C_s}{\tau} + \sum_i \frac{C_{oi}}{M_i \tau} + \sum_i \frac{D_i M_i \tau C_{1i}}{2}$$

Apart from these models, there are two other models by Darwish and Odah [5] and Verma et al. [10] that study the joint replenishment problems for multiple retailers under VMI in the presence

of penalty scheme and the same models will be used for comparison purpose with our proposed model.

4. Equivalence of the proposed model and JELS model with URI

In this section we establish the equivalence between our proposed model and JELS model under unequal replenishment intervals.

The constraint (i) in the proposed model can be rearranged as follows:

$$y_i p_i \geq \frac{C_{oi}}{M_i \tau} + \frac{D_i M_i \tau C_{li}}{2} - \frac{D_i C_{oi}}{Q_i} - \frac{Q_i C_{li}}{2} \quad \forall i \text{ which acts as the side payment from the supplier}$$

to each of the retailers to compensate them for carrying extra inventory. The same constraint will be binding in nature, as the supplier will always prefer to pay the retailers as less as possible since any payment more than the least value of $y_i p_i$ would reduce the gains of the supplier.

$$\text{Thus we can write: } y_i p_i = \frac{C_{oi}}{M_i \tau} + \frac{D_i M_i \tau C_{li}}{2} - \frac{D_i}{Q_i} C_{oi} - \frac{Q_i}{2} C_{li} \quad \forall i$$

$$\Rightarrow \sum_i y_i p_i = \sum_i \frac{C_{oi}}{M_i \tau} + \sum_i \frac{D_i M_i \tau C_{li}}{2} - \sum_i \frac{D_i}{Q_i} C_{oi} - \sum_i \frac{Q_i}{2} C_{li}$$

Replacing $\sum_i y_i p_i$ in the objective function of the proposed model, we get the following new objective function (Constant terms have been excluded from the objective function as they don't influence the final decision) given as:

$$\text{Min } \frac{C_s}{\tau} + \sum_i \frac{C_{oi}}{M_i \tau} + \sum_i \frac{D_i M_i \tau C_{li}}{2}$$

This is same as the objective function under the JELS model with URI, proving the equivalence of the two in terms of overall costs.

In the next section we provide a comparison between our model and other joint replenishment models through numerical an example.

5. Numerical Results

We consider a setting where 1 supplier is supplying to 4 retailers. The data for the problem setting is given in table 1. In order to solve our proposed model and all the other models, we have used LINGO 13 software on an Intel Core 2 Duo 2.6 GHz PC with 2 GB RAM.

Table 1: Data used for the comparison

Retailer data						Supplier data
Retailer	D_i (Demand)	C_{Ii} (Inventory holding cost)	C_{Oi} (order cost)	EOQ	x_i (per unit penalty)	C_s (Setup cost)
R1	400	0.8	40	200.0	0.8	120
R2	1000	0.8	45	335.4	0.8	
R3	8000	1	50	894.4	1	
R4	18000	1	60	1469.7	1	

For the purpose of comparison with the earlier mentioned four classes of replenishment models, we consider total supply chain cost (TSC) which includes the following:

1. Supplier's setup cost
2. Ordering cost for all the retailers
3. Inventory carrying cost for all the retailers

The results corresponding to the proposed model and that of the other models are given in table 2 and table 3 respectively.

Table 2: Results corresponding to the proposed model

Total SC Cost	τ		
3890.6	0.131		
z_1	z_2	z_3	z_4
200	335.4	894.4	1469.6
q_1	q_2	q_3	q_4

210	393	1048	2358
M_1	M_2	M_3	M_4
4	3	1	1

Table 3: Results corresponding to other models.

Models	Total SC Cost	τ		
VMI model with ERI [5]	4205.48	0.1265		
VMI model with URI [10]	3926.76	0.1143		
JELS model with ERI	4133.47	0.1520		
JELS model with URI	3890.60	0.1310		
	q_1	q_2	q_3	q_4
VMI model with ERI [5]	50.6	126.5	1011.9	2276.8
VMI model with URI [10]	182.9	343.0	914.4	2057.4
JELS model with ERI	61.0	152.41	1219.3	2743.4
JELS model with URI	209.6	393.0	1048.0	2358.0
	M_1	M_2	M_3	M_4
VMI model with ERI [5]	1	1	1	1
VMI model with URI [10]	4	3	1	1
JELS model with ERI	1	1	1	1
JELS model with URI	4	3	1	1

NOTE: - Darwish and Odah [5] consider equal reorder interval policy under VMI with EOQ set as the upper limit for the retailers. In their model the supplier carries inventory. However, for the data taken in table 1, the optimal solution of their model results in supplier carrying no inventory and hence is appropriate for the sake of comparison. Verma et al. [10] consider URI model under VMI with EOQ as the upper limit for the retailers.

It can be seen from the above tables that the proposed model performs better than the models of JELS with ERI, VMI model with ERI [5] and VMI model with URI [10]. Moreover, the results for the proposed model and JELS model with URI are identical. In the next subsection we carry out sensitivity analysis by varying some parameters and then provide a further comparison of the various models.

5.1. Sensitivity Analysis

In a single supplier multiple retailer scenario under VMI, it is interesting to see how the individual retailer's cost gets affected by various conditions. Specifically, in this section we examine how the cost of a retailer gets impacted by a) the presence of a big retailer and b) the change in its ordering cost. Apart from the proposed model, the sensitivity analysis is performed for the other multiple retailer VMI models viz., Darwish and Odah [5] and Verma et al. [10].

5.1.1. Impact on the individual retailer's cost in the presence of a big retailer

The data used for the sensitivity analysis is as shown in table 4.

Table4: Data for the sensitivity analysis 5.1.1.

Retailer data						Supplier data
Retailer	D_i (Demand)	C_{hi} (Inventory holding cost)	C_{oi} (order cost)	EOQ	x_i (per unit penalty)	C_s (Setup cost)
R1	400	0.8	40	200.0	0.5	120
R2	1000	0.8	40	316.22	0.5	
R3	3000	0.8	40	547.72	0.5	
R4	D4	0.8	40	-	0.5	

Retailer R4 is considered to be the big retailer in terms of its annual demand. The demand for R4 (D4) is varied from 8000 to 57000 through 99 instances. All the other parameters are kept same for all the retailers. We measure the impact of varying demand of the big retailer on the smaller retailers by the percentage deviation in their costs from the ideal scenario under their respective EOQ. The results for the sensitivity analysis can be summarized as follows :-

As the size (annual demand) of the retailer R4 increase from 8000 to 57000

1. Darwish and Odah [5] :- Smallest retailer (R1) suffers the most, followed by the second smallest retailer (R2) and so on.
2. Verma et al. [10] :- Largest retailer (R4) suffers the most, followed by the second largest retailer (R2) and so on.
3. Proposed model :- All the retailers remain insensitive to the change in the size of R4.

Unlike other models, the proposed model is such that every retailer (irrespective of its size) remains in the VMI partnership willingly.

5.1.2. Impact on the individual retailer’s cost by the variation in its ordering cost

The data used for the sensitivity analysis is as shown in table 5.

Table 5: Data for the sensitivity analysis 5.1.2.

Retailer data				Supplier data
Retailer	D_i (Demand)	C_{li} (Inventory holding cost)	x_i (per unit penalty)	C_s (Setup cost)
R1	400	0.8	0.5	120
R2	1000	0.8	0.5	
R3	3000	0.8	0.5	
R4	8000	0.8	0.5	

We aim to study the impact on individual retailer’s cost when its ordering cost is varied from 5 to 100 through 20 instances. While we vary the ordering cost for one of the retailers, the ordering cost for all the other retailers is kept at 40. Specifically, we study the impact on the smallest (R1) and the biggest (R4) retailer by performing the analysis independently on both. We measure the impact by the percentage deviation in the retailer’s cost from the ideal scenario under the EOQ. The results for the sensitivity analysis can be summarized as follows :-

As the ordering cost increases from 5 to 100

1. Darwish and Odah [5] :- Percentage change in the smallest retailer’s cost is positive and it increases linearly. For the largest retailer, the percentage change is negative and it decreases non linearly.
2. Verma et al. [10] :- Percentage change in the smallest retailer’s cost is almost zero. For the largest retailer, the percentage change is negative and it decreases non linearly.
3. Proposed model :- Irrespective of the size, percentage change in the retailer’s cost is zero.

Unlike other models, all the retailers irrespective of their size remains unaffected by the change in their ordering cost.

6. Conclusion

In this paper, we develop a model for determining the optimal replenishment policy in the context of single supplier multiple retailer scenario under VMI. It has been found that the proposed model outperforms the existing models in the literature. The contributions of the paper can be summarized in the following points:-

- 1) We provide the optimal replenishment policy in the context of single supplier–multiple retailer scenario under VMI.
- 2) The JELS model is known to provide the optimal replenishment policy in the context of single supplier – single retailer scenario. In this paper, we show that the same is also true for single supplier – multiple retailer scenario. Further, we also establish the equivalence of the proposed model with the JELS model.
- 3) In a multiple retailer situation two types of replenishment policy arise, equal and unequal replenishment interval. Through our proposed model, we reaffirm the point that URI policies being more generalized outperform the ERI policies.
- 4) Through the sensitivity analysis we show that the proposed model is much more robust than the other models. We find (for the proposed model) that the individual retailers remain unaffected by various factors such as presence of the bigger retailers and the change in the ordering cost.

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